Nuclear Theory - Course 227

SOURCE NEUTRON EFFECTS

The source neutrons available in CANDU reactors are those from spontaneous fission and those from the photoneutron reaction with deuterium. As mentioned in lesson 227.00-2, spontaneous fission produces a neutron flux which is approximately 10^{-12}% of the full power flux. The strength of the photoneutron source depends on the number and energy of the photons present. At significant power levels (>10%) the gamma flux is directly proportional to the power level; thus, the photoneutron reaction produces a photoneutron flux which is proportional to the total neutron flux present. At low power levels and particularly when shutdown, the strength of the photoneutron source depends on the inventory of fission products which produce the high energy (>2.2 MeV) gammas required for the photoneutron reaction. The longest lived relevant fission product decay chain has a half life of about 15 days; thus the photoneutron source persists for several weeks after shutdown, decreasing essentially exponentially from approximately 10^{-5}% of full power one day after shutdown. As we shall see shortly the values given for the actual neutron fluxes in the reactor due to the photoneutron source do not include any fission multiplication of this source.

The Effect of Neutron Sources on the Total Neutron Population

In a critical reactor with no neutron sources other than induced fission, the neutron population in the reactor remains constant from one generation to the next. In other words, neutron losses due to absorption and leakage exactly take care of the excess neutrons generated by fission that are not required to keep the chain reaction going.

Now imagine a neutron source emitting S_0 neutrons in each neutron generation time to be inserted into the reactor, and let this reactor be subcritical with a value of k just less than 1.

The number of neutrons present at the end of the first generation will be S_0, because that is how many are emitted in that time. At the end of the second generation, these S_0 will have become S_0k neutrons, and another S_0 neutrons will have been added by the source, giving us a total of $S_0 + S_0k$. At the end of the third generation these $S_0 + S_0k$ neutrons will have become $(S_0 + S_0k)k$ neutrons, and again another S_0 neutrons will have been added by the source to give us a grand total of $S_0 + S_0k + S_0k^2$.

If you pursue this sort of argument indefinitely, you will appreciate that we are going to end up with a final neutron population, S_∞, given by

$$S_\infty = S_0 + S_0 k + S_0 k^2 + S_0 k^3 + S_0 k^4 + ----$$

$$= S_0 (1 + k + k^2 + k^3 + k^4 + -----)$$

With k less than one, the sum

$$(1 + k + k^2 + k^3 + k^4 + -----) = \frac{1}{1-k}$$

We can therefore say that

$$S_\infty = \frac{S_0}{1-k} \qquad (1)$$

You can see that expression seems to be quite a reasonable one, because you can say if S_∞ is our final population, it will become $S_\infty k$ after one more generation. If k is less than 1, this means that $S_\infty - S_\infty k$ neutrons have been removed and these are made up by S_0 new ones emitted.

For example if $S_\infty = 5000 \frac{\text{neutrons}}{\text{generation}}$ and $k = .8$, these 5000 neutrons will become kS_∞ or 4000 neutrons in the next generation and the source S_0 neutrons per generation time will make up the cycle losses ($S_\infty - kS_\infty$) of 1000 $\frac{\text{neutrons}}{\text{generation}}$. Thus;

$$S_0 = S_\infty - kS_\infty$$

or

$$S_\infty = \frac{S_0}{1-k} \qquad (1)$$

By observing that;

$$1 - k = -\Delta k$$

we can rewrite equation (1) as;

$$S_\infty = -\frac{S_0}{\Delta k} \qquad (2)$$

Equation (1) and (2) are equivalent and you may use whichever one is convenient.

While we have developed equations (1) and (2) in terms of S_∞ and S_0 in neutrons per generation, the equations are equally valid in terms of power in watts or percent of full power as was shown in lesson 227.00-8.

Subcritical Multiplication

It is important to realize that even in a reactor that is well below critical (eg, -40 mk, typical of a reactor trip) the equilibrium source level (S_∞) is 25 times greater than the actual photoneutron source.

$$S_\infty = \frac{S_0}{1 - .96} = \frac{S_0}{.04} = 25\ S_0$$

The factor $(\frac{1}{1-k})$ can be called the *subcritical multiplication factor*. In the example above, the subcritical multiplication factor is 25. Thus, the indicated source level (S_∞) is 25 times the actual source level (S_0). This means that fission is producing 25 times as many neutrons as the source. The amount of subcritical multiplication depends only on the value of k. For example, if we used only half of the shutoff rods used in the previous example, such that we had -20 mk:

$$S_\infty = \frac{1}{1 - .98}\ S_0 = 50\ S_0$$

Now the subcritical multiplication factor is 50.

In a subcritical reactor without a neutron source the neutron population would totally collapse; however, when a source is present it is not the major constituent of the neutron population (provided $k>0.5$).

Calculation of k in a Subcritical Reactor

Suppose a reactor is shutdown with a constant indicated power of $2 \times 10^{-5}\%$. The operator inserts +1 mk by withdrawing an adjuster and power stabilizes at $3 \times 10^{-5}\%$. Find the original value of k.

For the first case before the reactivity addition;

$$P_\infty = \frac{P_0}{1-k_i}$$

$$2 \times 10^{-5}\% = \frac{P_0}{1-k_i}$$

After the reactivity addition;

$$3 \times 10^{-5}\% = \frac{P_0}{1 - (k_i + .001)}$$

Since P_0 can be assumed to be the same in both cases, the equations may be solved for k_i.

$$P_0 = (1 - k_i) \times 2 \times 10^{-5}\%$$

$$P_0 = [1 - (k_i + .001)] \times 3 \times 10^{-5}\%$$

$$2 \times 10^{-5}\%(1 - k_i) = 3 \times 10^{-5}\%(.999 - k_i)$$

$$2 - 2\,k_i = 2.997 - 3\,k_i$$

$$\underline{\underline{k_i = .997}}$$

You can always find the value of k in a shutdown reactor by changing reactivity, noting the power before and after a change and doing a simple calculation.

Time Considerations

In a subcritical reactor the power will increase to a new equilibrium value each time positive reactivity is added. The magnitude of the increase and the time it takes for power to stabilize will depend on the value of k. The closer k is to one the larger the power increase for a given reactivity increase, and the longer the time for power to stabilize. This is demonstrated in Figure 1 where we start with $P_\infty = 1 \times 10^{-5}\%$ and $k = 0.90$ and add reactivity in +10 mk steps allowing P_∞ to stabilize after each reactivity addition.

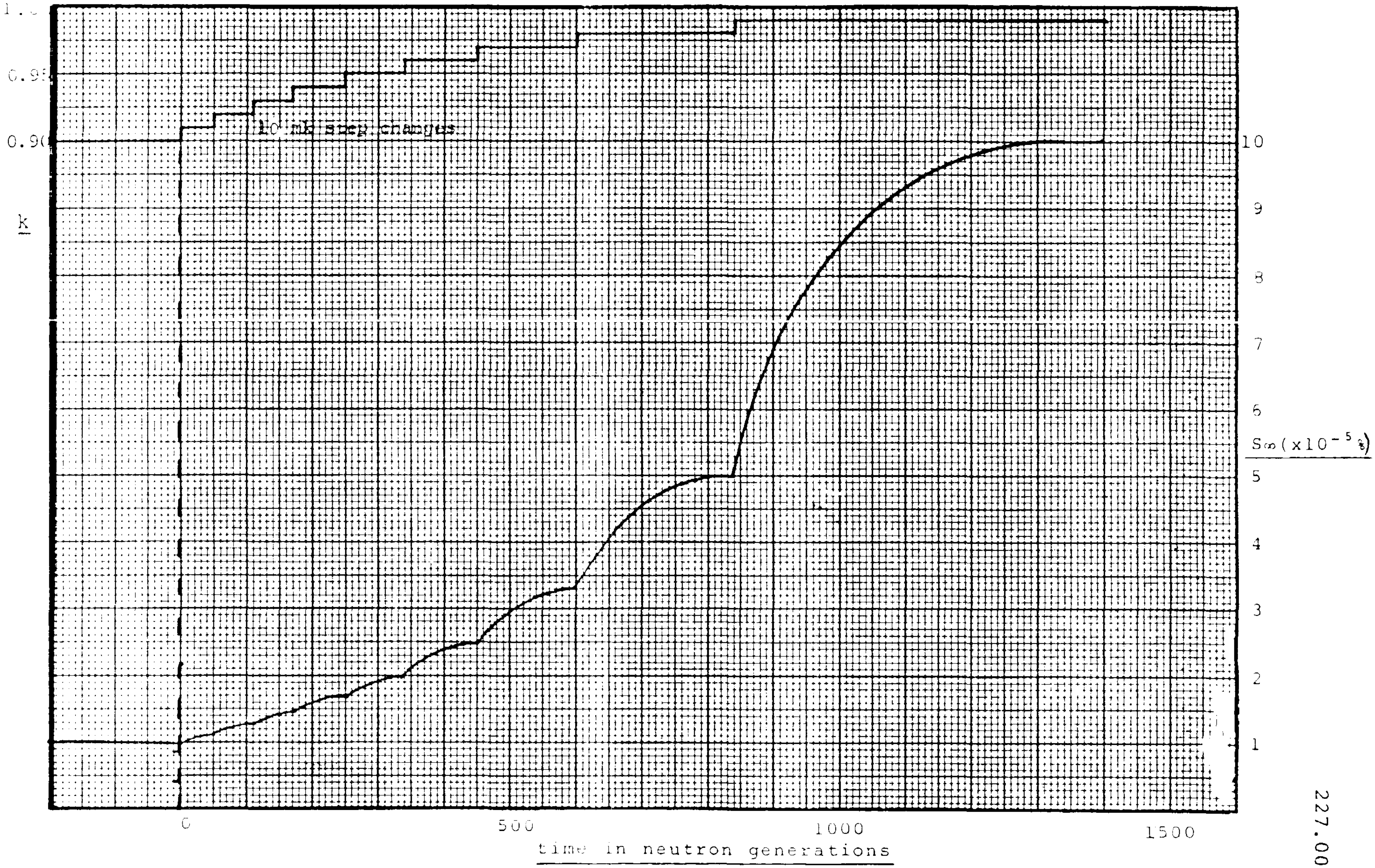

FIGURE 1

Power increase in a subcritical reactor.

Justification for relative magnitudes of the power increases can be done in a straightforward manner using equation (1) or (2) and is left to the student. The time consideration is somewhat more obscure. Assume we start with a source $S_0 = 1000 \dfrac{\text{neutrons}}{\text{generation}}$ at time zero and ask the question, "How long will it take to reach equilibrium (S_∞)? Equilibrium will be obtained when the effect of the first group of S_0 neutrons has totally disappeared (ie, $S_0k^N = 0$ or $k^N = 0$, where N is the number of neutron generations). For example, we will compare the number of generations required for the first group of source neutron to disappear with k = 0.8, 0.9, and 0.95. The tabulated values are S_0k^N.

k \ N	1	2	3			30		62		127
0.80	800	640	512			1		-		-
0.90	900	810	729			42		1		-
0.95	950	903	857			215		42		1

As you see, with k = 0.8 it would take about 30 generations to reach equilibrium, with k = 0.9 about 62 generations, and with k = 0.95 about 127 generations. (Strictly of course it takes an infinite time, this example gets to within 0.1%.)

Effect of Sources when $k \geq 1$

When the reactor is critical, equations (1) and (2) don't apply because they are based on the assumption that the series $1 + k + k^2 + k^3 + k^4$ ----) has a finite sum. When constant, and S_0 new ones will be added every generation, so that the neutron population will then merely increase indefinitely and at a constant rate of S_0 neutrons for every generation.

This rate of increase is insignificant if the reactor power is greater than $\sim 10^{-2}$% and would generally be obscured by the automatic regulation of the reactor. In a supercritical reactor any effects of the sources may be ignored.

ASSIGNMENT

1. Explain why the total neutron population in a subcritical reactor is significantly higher than the source level.

2. Define the subcritical multiplication factor.

3. Explain why it is possible to have a constant fission rate in a subcritical reactor.

4. Assuming $S_0 = 1000 \frac{\text{neutrons}}{\text{generation}}$ and the average neutron lifetime is 0.1 seconds, how long will it take to reach equilibrium when k = 0.999? (You need a calculator for this problem.)

J.E. Crist
A. Broughton